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# **Brake Testing Methodology Study - Driver Effects Testing**

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16. Abstract  <p>The National Highway Traffic Safety Administration (NHTSA) is exploring the feasibility of developing brake tests to measure brake system performance of light vehicles.</p> <p>Developing test procedures requires controlling test variability so that measured differences between vehicles are more than just experimental noise. Possible sources of uncontrolled variability include environmental conditions, vehicle-to-vehicle differences for a given model, brake system changes with time, test driver differences, test surface friction changes with time, and test surface friction differences between test sites.</p> <p>The objective of this project was to determine the level of variability in stopping distance tests of light vehicles that is due to differences between drivers. 648 stopping distance tests were conducted with three expert drivers in three different cars on wet and dry asphalt with the ABS working and disabled.</p> <p>All four independent factors had a statistically significant effect on the stopping distance. Several interactions of the independent variables were also significant. The largest main effect was the differences between vehicles. The effect of ABS and surface condition and drivers were all fairly small. The interaction between ABS condition and surface was also small but indicated that ABS mattered more on the wet surface. The remaining interactions were also fairly small.</p> <p>In conclusion, this test procedure measured differences between all three vehicles with very high statistical certainty. Unfortunately, this test procedure used a large number of replications and drivers. If the goal for a brake test program is only to distinguish large differences in braking performance, fewer replications and drivers would be needed for each vehicle tested.</p>			
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The testing performed in this study was for methodology development. The vehicles tested were used vehicles leased from local automobile dealerships or vehicles owned by the National Highway Traffic Safety Administration. The performance of these vehicles may not be indicative of new vehicles.

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## **1.0 INTRODUCTION**

The National Highway Traffic Safety Administration (NHTSA) is exploring the feasibility of developing brake tests to measure brake system performance of light vehicles.

Random variability in brake testing can be quite large for light vehicles. Developing brake system performance tests requires controlling test variability so that measured differences between vehicles is more than just experimental noise. Possible sources of variability include environmental conditions, vehicle-to-vehicle differences for a given model, brake lining conditioning, test driver differences, test surface friction changes with time, test surface friction differences between test sites.

In 1997, eleven vehicles were tested at NHTSA's Vehicle Research and Test Center (VRTC). The goals were to evaluate various brake system performance measures and to quantify the levels and sources of variability associated with the measures<sup>1</sup>.

In 1998, a second test program was conducted to further explore brake testing variability. The objective of this program was to determine the level of variability in stopping distance tests of automobiles that is due to differences between drivers for a variety of conditions. VRTC conducted tests with three expert test drivers in three different cars on wet and dry asphalt with the anti-lock brake systems (ABS) working and disabled. This report documents that test program.

## **2.0 TEST METHOD**

### **2.1 The Vehicles and Preparation**

VRTC leased three used late model cars for testing. The vehicles were commonly available small, medium, and large passenger cars. All the vehicles were equipped with four wheel disk brakes and ABS. A list of the vehicles tested is shown in Table 2.1. Complete information on the vehicles can be found in Appendix A.

Table 2.1 -- Test Vehicles

Vehicle Category	Vehicle	ABS Supplier	Test Weight
Small Automobile	1995 Chrysler Neon	Allied Signal	1234 kg
Mid-Size Automobile	1996 Ford Taurus	Bosch	1710 kg
Large Automobile	1997 Lincoln Town Car	ITT	1941 kg

VRTC rebuilt each vehicle's brake system with new original equipment brake linings, brake rotors, and tires. The brake fluid was replaced with new brake fluid. Also, any other components which might affect the braking performance that appeared to be worn were replaced so each vehicle's brake system was in "like new" condition. The brake systems were then burnished according to Federal Motor Vehicle Safety Standard (FMVSS) 135<sup>2</sup>. The vehicles were tested with only the driver and instrumentation onboard, which is commonly called lightly loaded (LLVW).

## **2.2 Instrumentation and Data Collection**

Each vehicle was equipped with a Tracktest fifth wheel and Labeco Performance Monitor Model 625. These provided vehicle speed at the beginning of braking and total stopping distance and were triggered by the brake light switch. The initial speed and stopping distance data from the fifth wheel, and whether there was any wheel lock-up during the stop were manually recorded by the driver. Other instrumentation included a brake pedal force transducer and brake lining thermocouples. Each of these had a readout for the driver to use during test execution. The brake lining temperature prior to the beginning of each stop and maximum brake pedal force after each stop were also manually recorded by the driver.



### **2.3 Test Conduct**

Stopping distance tests were conducted according to the FMVSS 135 test procedure to the extent possible. This included a brake pedal force limit of 500 N. Also, the brake lining temperature prior to each stop was required to be between 65 and 100° C. The stops were in a straight line and began at 100 km/h. The drivers were instructed to achieve the shortest stopping distance possible within the pedal force limits and with no wheel lock-up in the cases where the ABS was disabled. FMVSS 135 tests are typically conducted on dry concrete. However, these tests were conducted on asphalt so that the results from this program would be more comparable to the 1997 braking test program. It is believed that for the purposes of this study, the difference between dry asphalt and dry concrete is minimal.

The vehicles were tested under a variety of conditions to extend the applicability of the results of the program. The vehicles were tested on two surface conditions and two brake conditions. The vehicles were tested on both dry and wet asphalt to see if driver effects were different on high and medium coefficient of friction surfaces. These two test surfaces were selected because they have more stable coefficients of friction over time than other surfaces like wet Jennite and epoxy. The tests were conducted at the Transportation Research Center (TRC) on the Vehicle Dynamics Area. The peak friction coefficient of the asphalt of the Vehicle Dynamics Area is regularly monitored by TRC with a skid trailer using an ASTM E1136 tire according to the ASTM E1337-90 test procedure. The measured nominal peak friction coefficients of the wet and dry asphalt during testing was 0.66 and 0.86 respectively.

The vehicles' anti-lock brake systems were disabled for some stops to simulate a vehicle without ABS. This would show if driver effects were different for vehicles not equipped with ABS. The ABS was disabled by removing the fuse for the system. For the ABS-disabled tests, stops were made with the driver modulating the brakes to achieve the shortest stop without locking any wheels and not exceeding the pedal force limit of 500 N. For stops conducted with ABS-on, the driver rapidly applied full pedal effort up to 500 N. The shortest of six or three stops respectively for each test

condition was the performance measure. Although the braking performance of an ABS-equipped vehicle with the ABS disabled may not be the same as a vehicle not equipped with ABS, the driver effects are assumed to be the same.

## **2.4 The Drivers**

The drivers who participated in the test program were all professional test drivers with varying amounts of experience. They are considered representative of the pool of test drivers that might be used in a brake testing program conducted at any automotive proving grounds in the United States.

Driver 1 had been a professional test driver at TRC for 6 years. For the previous 2 years Driver 1 had driven in test programs involving best-effort braking and maneuvers at the limit of vehicle handling with both heavy trucks and light vehicles.

Driver 2 had been a professional test driver at TRC for 18 years. For the last 12 years Driver 2 had driven in test programs involving best-effort braking and maneuvers at the limit of vehicle handling with both heavy trucks and light vehicles.

Driver 3 had been a professional test driver at TRC for 13 years. For the last 8 years Driver 3 had driven in test programs involving best-effort braking and maneuvers at the limit of vehicle handling with both heavy trucks and light vehicles.

## **2.5 Experimental Design**

A split plot factorial experimental design was chosen. In such a design the independent factors consist of what are referred to as “between” factors and “within” factors<sup>3</sup>. In this study, the independent factors and their levels were:

Between factors and levels

- Driver (Drivers 1,2,3)
- Replications within a driver (Replications 1,2,3,4)

Within factors and levels

- Vehicle (Neon, Taurus, Town Car)
- ABS condition (on/off)
- Surface condition (dry/wet)

In a split plot factorial design each level of a between factor receives all combinations of the within factors. Thus each driver drove each vehicle under every combination of ABS and surface condition.

The design and all the factors are shown in Table 2.2. Drivers, vehicles, ABS condition, and surface condition are treated as fixed factors (or effects) and replications are treated as a random factor. A fixed effect is an independent factor for which all levels about which statistical inferences are to be drawn are included in the study. A random factor is an independent factor for which levels in a study are a random sample from a larger population.

Neither the drivers nor vehicles included in this study were from a random sample of the population of all vehicles and drivers. The drivers were selected for inclusion in the study based on availability and having at least a few years of driving in test programs that involved braking at the vehicle limit. The vehicles were selected based on availability and size. That is, the most readily available small, mid-size, and large cars were used. Therefore, conclusions about the drivers and vehicles in this study cannot be extended to other levels (i.e. other models of vehicle or other drivers) on strictly statistical grounds. However, conclusions about the drivers and vehicles can be extended to levels outside those included in the study based on logic and engineering judgement of the representativeness of the levels included in the study.

Table 2.2 -- Partial Test Matrix

		Replication	Treatment Combination					
			Vehicle 1 ( $v_1$ ) ABS-on ( $b_1$ ) Surface dry ( $s_1$ )	Vehicle 1 ( $v_1$ ) ABS-on ( $b_1$ ) Surface wet ( $s_2$ )	Vehicle 1 ( $v_1$ ) ABS off ( $b_2$ ) Surface dry ( $s_1$ )	Vehicle 1 ( $v_1$ ) ABS off ( $b_2$ ) Surface wet ( $s_2$ )	...	Vehicle 3 ( $v_3$ ) ABS off ( $b_2$ ) Surface wet ( $s_2$ )
Driver	1	1	$d_1v_1b_1s_1$	$d_1v_1b_1s_2$	$d_1v_1b_2s_1$	$d_1v_1b_2s_2$	...	$d_1v_3b_2s_2$
		2	:	:	:	:	...	:
		3	:	:	:	:	...	:
		4	$d_1v_1b_1s_1$	$d_1v_1b_1s_2$	$d_1v_1b_2s_1$	$d_1v_1b_2s_2$	...	$d_1v_3b_2s_2$
	2	1	$d_2v_1b_1s_1$	$d_2v_1b_1s_2$	$d_2v_1b_2s_1$	$d_2v_1b_2s_2$	...	$d_2v_3b_2s_2$
		2	:	:	:	:	...	:
		3	:	:	:	:	...	:
		4	$d_2v_1b_1s_1$	$d_2v_1b_1s_2$	$d_2v_1b_2s_1$	$d_2v_1b_2s_2$	...	$d_2v_3b_2s_2$
	3	1	$d_3v_1b_1s_1$	$d_3v_1b_1s_2$	$d_3v_1b_2s_1$	$d_3v_1b_2s_2$	...	$d_3v_3b_2s_2$
		2	:	:	:	:	...	:
		3	:	:	:	:	...	:
		4	$d_3v_1b_1s_1$	$d_3v_1b_1s_2$	$d_3v_1b_2s_1$	$d_3v_1b_2s_2$	...	$d_3v_3b_2s_2$

There were three levels of drivers and vehicles, two levels of ABS condition and surface condition, and four replications. This produced 36 cells, with four replications in each cell, and a total of 144 possible data points. Since there are three stops for every data point with ABS-on and 6 stops for every data point with ABS-off, there was a total of 648 stopping distance tests performed for this program. As previously stated, the dependant variable in this study was the shortest stopping distance of the three or six stops.

## 2.6 Procedure

The run order of the experiment for Driver 1 is shown in Table 2.3. As can be seen, the order of the test conditions was randomized within each vehicle and across replications. For simplicity, all the drivers tested the same ABS and asphalt condition at the same time in one of the three vehicles. For example, in replication 1, Driver 1 drove the Taurus, Driver 2 the Town Car, and Driver 3 the Neon. They simultaneously conducted three stops each on dry asphalt with the ABS-on, then three stops each on wet asphalt with the ABS-on, then six stops each on dry asphalt with the ABS-off, then six stops each on wet asphalt with the ABS-off. Then the drivers changed cars and repeated all four test conditions in a different order. Then all the test conditions were again repeated in different cars, completing one replication. All four replications were run in this manner, completing the data collection. The goal of having the drivers testing the same conditions simultaneously was to reduce the likelihood of procedural error. This test procedure also insured that effects due to environmental conditions, time of day, driver fatigue or learning canceled out.

Table 2.3 -- Run Order for Driver 1

Replication 1 of 4	Replication 2 of 4	Replication 3 of 4	Replication 4 of 4
Taurus	Taurus	Taurus	Taurus
ABS-on, dry	ABS-off, wet	ABS-on, wet	ABS-off, dry
ABS-on, wet	ABS-off, dry	ABS-on, dry	ABS-off, wet
ABS-off, wet	ABS-on, dry	ABS-off, dry	ABS-on, wet
ABS-off, dry	ABS-on, wet	ABS-off, wet	ABS-on, dry
Town Car	Town Car	Town Car	Town Car
ABS-off, dry	ABS-on, wet	ABS-off, wet	ABS-on, dry
ABS-off, wet	ABS-on, dry	ABS-off, dry	ABS-on, wet
ABS-on, wet	ABS-off, dry	ABS-on, dry	ABS-off, wet
ABS-on, dry	ABS-off, wet	ABS-on, wet	ABS-off, dry
Neon	Neon	Neon	Neon
ABS-on, wet	ABS-off, wet	ABS-on, dry	ABS-off, dry
ABS-on, dry	ABS-off, dry	ABS-on, wet	ABS-off, wet
ABS-off, wet	ABS-on, wet	ABS-off, dry	ABS-on, dry
ABS-off, dry	ABS-on, dry	ABS-off, wet	ABS-on, wet

## **2.7 Data Reduction**

As previously stated, the target test speed was 100 km/h. The stopping distances were corrected for minor variations in the speed from which the stop was initiated using the following formula from SAE J299<sup>4</sup>:

$$SD_{cor} = SD_{test} \times \frac{V_{target}^2}{V_{test}^2}$$

where:

$SD_{cor}$  = Corrected stopping distance

$SD_{test}$  = Actual stopping distance

$V_{test}$  = Actual test speed

$V_{target}$  = Target test speed

Note that SAE J299 states this speed correction formula is accurate only for speed differences up to 3.2 km/h. All stops in this study were initiated within that range. Once the stopping distance was corrected for speed, the shortest stop without wheel lock-up and with pedal force below 500 N for each test condition and replication was selected.

Then the corrected stopping distance data were analyzed using the statistical software packages Statistical Analysis System(SAS)<sup>5</sup> and Minitab<sup>6</sup>. Inferential tests of significance of the main effects and interactions was based on the Analysis of Variance (ANOVA) performed in SAS using the General Linear Models Procedure (proc GLM). See Appendix B for details of the model.

## **3.0 RESULTS**

### **3.1 Data Analysis**

Figure 3.1 shows all of the data from the experiment. Each point on the graph represents the best of three or six stops, depending on ABS condition, as previously described. As can be seen, not all

of the test conditions appear to have four data points from the four replications. This can be for two reasons. One reason is that the stopping distance from two replications for a driver in a set of test conditions has the same value. Overprinting makes these data points appear as one data point. Another more common reason is because in one or more of the replications, the driver did not achieve a stop without wheel lock-up and pedal force below 500 N in any of the three or six stops. This happened 3 times, all with Driver 1.

Preliminary plotting of the data showed a strongly right skewed data set. Figure 3.2 shows a histogram of all the data with a normal curve. The cause of the non-normality is that the laws of physics dictate that there is a lower limit for the stopping distance of each vehicle in each test condition. However, for each test there are any number of circumstances that can cause the stop to be longer, including variations in test surface friction, water depth, driver input, vehicle condition, environmental factors etc. The difficulty this causes in the data analysis is that for most statistical inferences (tests) to be valid, they must be made on normally distributed data. While ANOVA is generally robust to violations of normality<sup>7</sup>, transforming the data to give it a more normal distribution improves the accuracy of the statistical tests.

Several data transformations were tried on the data, including square root, reciprocal, natural logarithm, and base ten logarithm. The natural logarithm transformation was the best, bringing the data closest to a normal distribution. See Figure 3.3. As can be seen however, the data set is still somewhat right skewed. However, the statistical methods used are robust enough to not be effected by slightly skewed data. All statistical inferences were made on this transformed data set.

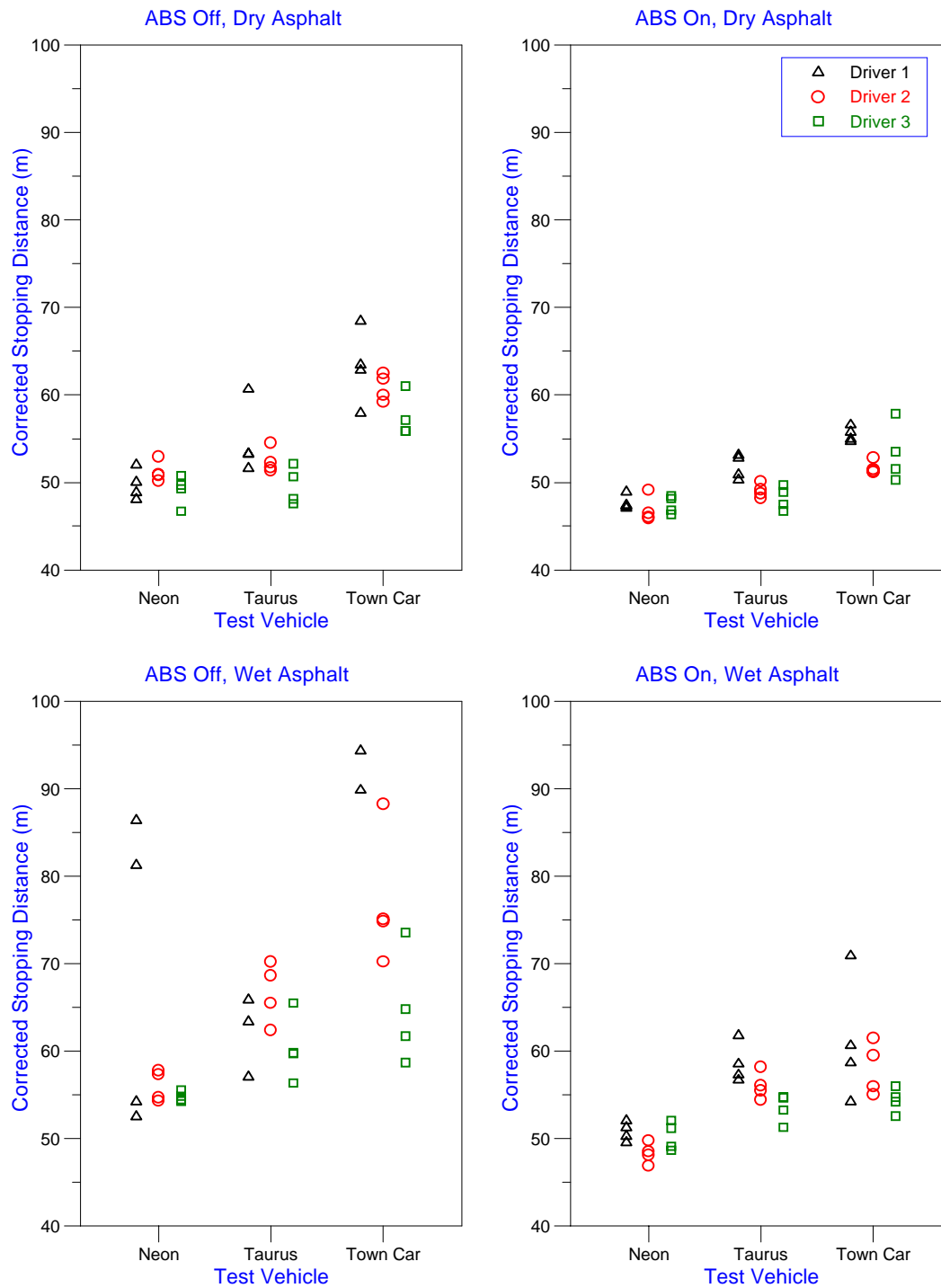


Figure 3.1 – Shortest Stopping Distance In Each Test Condition and Replication



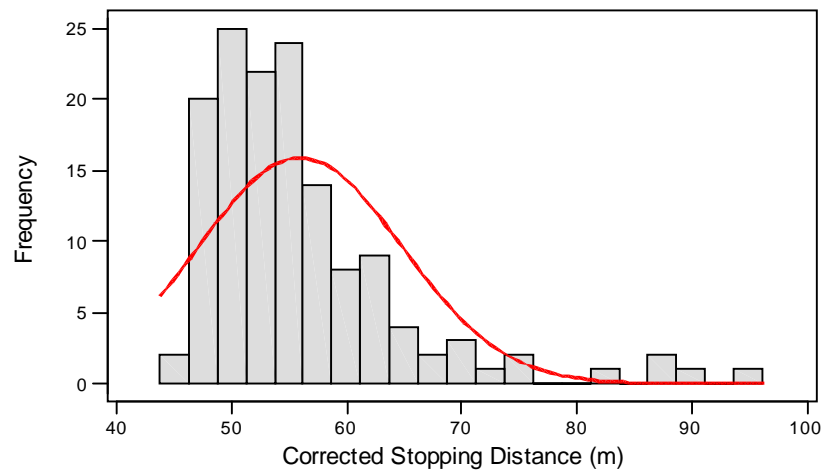


Figure 3.2 – Histogram of Corrected Stopping Distance Data

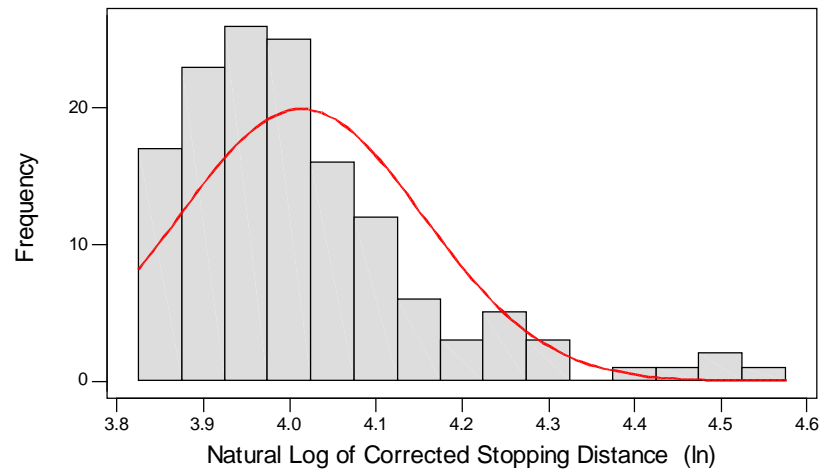


Figure 3.3 – Histogram of Natural Log Transformed Stopping Distance Data

The next step in the data analysis was to look at outliers in the natural log transformed data. Outliers are defined as data points with a standardized residual greater than 2.0 for a model that included all the independent variables and their two-way interactions. The outliers are shown in Table 3.1. First it was verified that the outliers were not data entry errors. Once the accuracy of the data points was verified, the driver data sheets for these stops were reviewed for any problems. Since nothing unusual was found in the driver data sheets, the distribution of the outliers was analyzed. The outliers were almost evenly distributed between all three vehicles. Also, nine of the outliers were with the ABS disabled and nine of the outliers were on wet asphalt. Additionally, eight of the outliers were from the least experienced driver. One would reasonably expect the most variation from the least experienced driver, with the ABS disabled, on the wet asphalt surface. The lack of compelling evidence to remove the outliers dictated that they remain in the data set.

Table 3.1 – Outliers in the Data

Driver	Vehicle	ABS Condition	Surface Condition	Stopping Distance(ln)	Fit (ln)	Standardized Residual (ln)
1	Taurus	Off	Dry	4.10	3.97	2.09
1	Neon	Off	Wet	3.96	4.14	-2.89
1	Taurus	Off	Wet	4.04	4.22	-2.86
1	Town Car	On	Wet	3.99	4.14	-2.30
1	Neon	Off	Wet	3.99	4.14	-2.39
1	Neon	Off	Wet	4.40	4.14	4.00
1	Town Car	Off	Wet	4.55	4.42	2.12
1	Neon	Off	Wet	4.46	4.14	4.97
2	Town Car	Off	Wet	4.48	4.33	2.42
3	Town Car	Off	Wet	4.07	4.21	-2.11

The next step was to plot the main effects. See Figure 3.4. A main effect is the average value of the dependent variable for an independent factor over all the other independent factors. The first box in Figure 3.4 shows the mean stopping distance for each driver over all the vehicles and test conditions.

The second box shows the mean stopping distance for each vehicle over all the drivers and test conditions. The third box shows the mean stopping distance for ABS-on and off over all the drivers, vehicles and test conditions. The fourth box shows the mean stopping distance for wet and dry asphalt over all the drivers, vehicles and ABS condition.

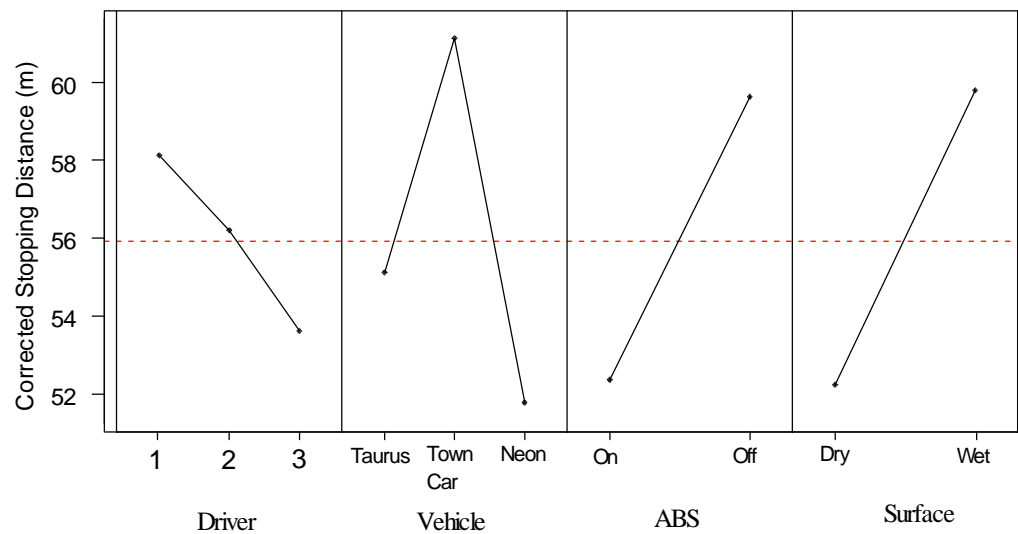


Figure 3.4 -- Main Effects Plot for Corrected Stopping Distance Data

As can be seen, the largest effect (difference in mean stopping distance) was between vehicles. The mean stopping distance of the Neon was about 10 meters shorter than the Town Car. The mean stopping distance of the Taurus was only 4 meters longer than the Neon. This result agrees with known frictional properties of tires, since one would expect the smallest car (least mass and tire load) to have the shortest stopping distance, the mid-sized car to have an intermediate stopping distance, and the large car (most mass and tire load) to have the longest stop. Factors like braking efficiency, brake balance, and ABS functioning also effect vehicle stopping distance.

The next largest effects were ABS condition and surface condition. The ABS-on mean stopping distance was about 7 meters shorter than ABS-off. The dry asphalt stopping distance was about 8 meters shorter than the wet asphalt. These results also are consistent with engineering principles.

The smallest effect was drivers. Driver 1 had the longest mean stopping distance of all the drivers. The difference between Driver 1 and 2 was only about 2 meters and the difference between Driver 2 and 3 is about 3m. The difference between Driver 1 and 3 is about 5 meters.

The next step of the data analysis was to plot the two-way interactions. See Figure 3.5. An interaction exists when the level of one independent factor changes the effect of another independent factor. Each box in Figure 3.5 shows whether there was an interaction between factors. The difference in slope between any two or three lines in a box indicates the amount of interaction between factors. If the lines are parallel in a box, there is no interaction between the factors. Similar to Figure 3.4, the y-axis is the mean corrected stopping distance in meters of each combination of factors.

The largest two-way interaction is between ABS condition and surface condition. This is shown in the lower right box in Figure 3.5. It shows that for the ABS-off stops, the effect of surface was greater. For the ABS-off condition, the mean stopping distance over all the vehicles and drivers was 54 meters for dry asphalt and 66 meters for wet. For the ABS-on condition, the mean stopping distance over all the vehicles and drivers was 50 meters for dry asphalt and 55 meters for wet asphalt. To put it in the simplest terminology, the box shows that ABS had a greater effect for stops on wet asphalt than dry asphalt, which matches expectations.

The next largest two-way interaction is between vehicle and ABS condition. The Taurus mean stopping distance over all the drivers and surface conditions was 53 meters for ABS-on and 55 meters for ABS-off. The Town Car mean stopping distance over all the drivers and surface conditions was 56 meters for ABS-on and 66 meters for ABS-off. In simplest terms, the Taurus was least affected by ABS condition, and the Town Car was most affected.

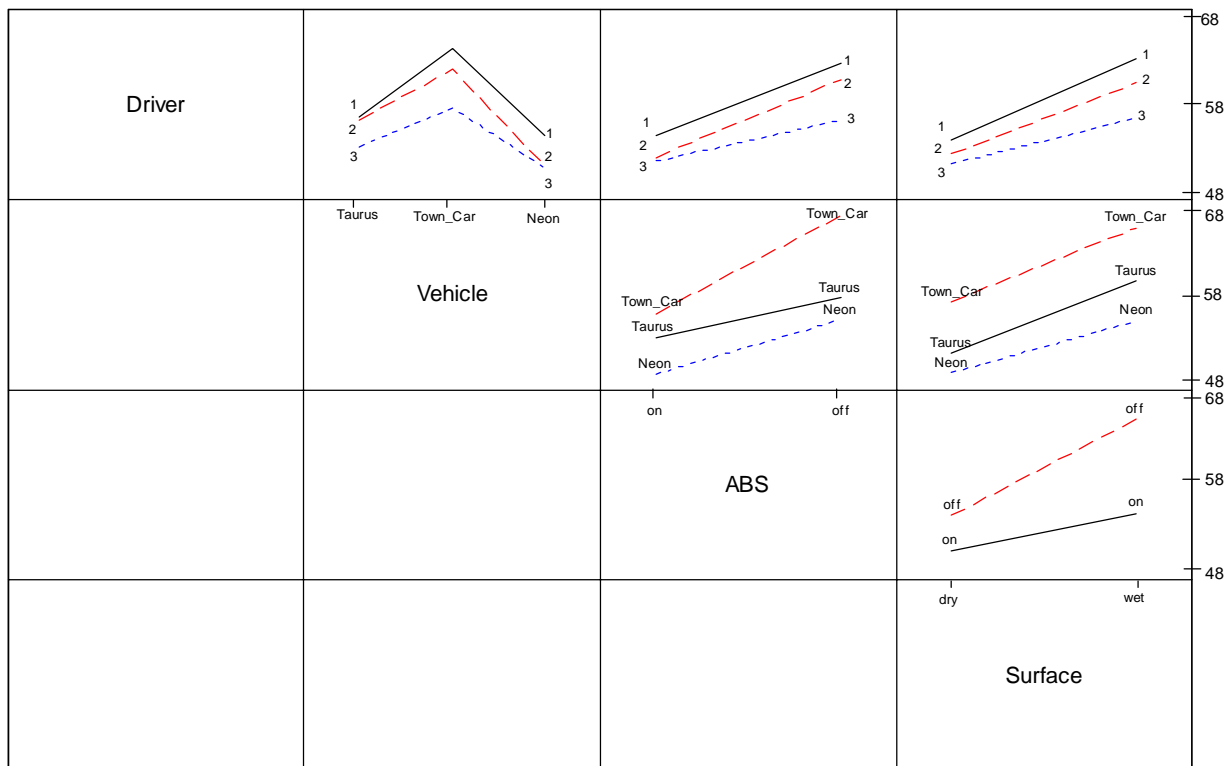


Figure 3.5 – Two-Way Interaction Plot of Corrected Stopping Distance Data

The other two-way interactions appear to be very small and so do not warrant further discussion at this point.

What is not known from just looking at the main effects plots and the two-way interactions plots is whether or not the differences they show are statistically significant.

Statistical significance implies that the measured result is unlikely to arise just by chance. Every experimental procedure generates data with some level of experimental noise. Testing for statistical significance is merely comparing the level of noise in the data with the supposed effect the independent variable has on the dependent variable. Throughout this analysis, the threshold for significance is that there is less than a 5% probability that the effect measured is really just experimental noise. This means that the probability is less than 5% a measured effect that was

deemed “significant” arose merely due to chance rather than because there truly is an effect. To test for the statistical significance of the main effects and interactions, the data was analyzed using an ANOVA suitable for the split-plot factorial design.

The model for the split-plot factorial design is<sup>3</sup>:

$$Y_{ijklm} = \mu + \alpha_j + \pi_{i(j)} + \beta_k + (\alpha\beta)_{jk} + (\beta\pi)_{ki(j)} + \gamma_l + (\alpha\gamma)_{jl} + (\gamma\pi)_{li(j)} + \delta_m + (\alpha\delta)_{jm} + (\delta\pi)_{mi(j)} + (\beta\gamma)_{kl} + (\alpha\beta\gamma)_{jkl} + (\beta\gamma\pi)_{kli(j)} + (\beta\delta)_{km} + (\alpha\beta\delta)_{jkm} + (\beta\delta\pi)_{kmi(j)} + (\gamma\delta)_{lm} + (\alpha\gamma\delta)_{jlm} + (\gamma\delta\pi)_{lmi(j)} + (\beta\delta\gamma)_{klm} + (\alpha\beta\delta\gamma)_{jklm} + (\beta\gamma\delta\pi)_{klmi(j)} + \varepsilon_{ijklm}$$

Where:

$Y_{ijklm}$  is the stopping distance in replication  $i$  and treatment combination  $ijklm$

$\mu$  is the grand mean stopping distance of all the tests

$\alpha_j$  is the treatment effect for driver  $j$

$\pi_{i(j)}$  is the block effect for replication  $i$

$\beta_k$  is the treatment effect for vehicle  $k$

$(\alpha\beta)_{jk}$  is the joint treatment effect of driver  $j$  and vehicle  $k$

$(\beta\pi)_{ki(j)}$  is the joint treatment effect for vehicle  $k$  and replication  $i$

$\gamma_l$  is the treatment effect for ABS condition  $l$

$(\alpha\gamma)_{jl}$  is the joint treatment effect for driver  $j$  and ABS condition  $l$

$(\gamma\pi)_{li(j)}$  is the joint treatment effect for ABS condition  $l$  and replication  $i$

$\delta_m$  is the treatment effect for surface condition  $m$

$(\alpha\delta)_{jm}$  is the joint treatment effect for driver  $j$  and surface condition  $m$

$(\delta\pi)_{mi(j)}$  is the joint treatment effect for surface condition  $m$  and replication  $i$

$(\alpha\beta\gamma)_{jkl}$  is the joint treatment effect for driver  $j$ , vehicle  $k$ , and ABS condition  $l$

$(\beta\gamma\pi)_{kli(j)}$  is the joint treatment effect for vehicle  $k$ , ABS condition  $l$ , and replication  $i$

$(\beta\gamma)_{kl}$  is the joint treatment effect for vehicle  $k$  and ABS condition  $l$

$(\beta\delta)_{km}$  is the joint treatment effect for vehicle  $k$  and surface condition  $m$

And:

$(\alpha\beta\delta)_{jkm}$  is the joint treatment effect for driver  $j$ , vehicle  $k$ , and surface condition  $m$   
 $(\beta\delta\pi)_{kmi(j)}$  is the joint treatment effect for vehicle  $k$ , surface  $m$ , and replication  $i$   
 $(\gamma\delta)_{lm}$  is the joint treatment effect for ABS condition  $l$  and surface condition  $m$   
 $(\alpha\gamma\delta)_{jlm}$  is the joint treatment effect for driver  $j$ , ABS condition  $l$ , and surface  $m$   
 $(\gamma\delta\pi)_{lmi(j)}$  is the joint treatment effect for ABS condition  $l$ , surface  $m$ , and replication  $i$   
 $(\beta\delta\gamma)_{klm}$  is the joint treatment effect for vehicle  $k$ , ABS condition  $l$ , and surface  $m$   
 $(\alpha\beta\delta\gamma)_{jklm}$  is the joint treatment effect for driver  $j$ , vehicle  $k$ , ABS  $l$ , and surface  $m$   
 $(\beta\gamma\delta\pi)_{klmi(j)}$  is the joint treatment effect for vehicle  $k$ , ABS  $l$ , surface  $m$ , and replication  $i$   
 $\epsilon_{ijklm}$  is the error term

This equation models each data point as the sum of the effect of all the independent factors and all the possible interactions of the independent factors and an error term. That is, the model includes the main effects, and two, three, and four-way interactions and an error term. Since none of the parameters in the model are empirically known, they are estimated from the data using the equations in Table 3.3 and 3.4. Table 3.2 is an aid to reading Tables 3.3 and 3.4.

Table 3.2 – Meaning of Symbols Used in Computational Formulas

Independent Factor	Effect	Number of Levels	Index
Replication	R	n	i
Driver	D	p	j
Vehicle	V	q	k
Brake	B	r	l
Surface	S	t	m

Table 3.3 – Split-Plot Factorial Design F-statistic Calculations<sup>3</sup>

Eqn. No	Computational Formulas	Degrees of Freedom	Error Eqn #
1	$ss\ b.\ bl=[DR]-[Y]$	$np-1$	N.A.
2	$ssD=[D]-[Y]$	$p-1$	3
3	$ssblD(D)=[DR]-[D]$	$p(n-1)$	N.A.
4	$ss\ w.\ bl=[DVBSR]-[DR]$	$np(qrt-1)$	N.A.
5	$ssV=[V]-[Y]$	$q-1$	7
6	$ssDV=[DV]-[D]-[V]+[Y]$	$(p-1)(q-1)$	7
7	$ssVxbl(D)=[DVR]-[DV]-[DR]+[D]$	$p(n-1)(q-1)$	N.A.
8	$ssB=[B]-[Y]$	$r-1$	10
9	$ssDB=[DB]-[D]-[B]+[Y]$	$(p-1)(r-1)$	10
10	$ssBxbl(D)=[DBR]-[DB]-[DR]+[D]$	$p(n-1)(r-1)$	N.A.
11	$ssS=[S]-[Y]$	$t-1$	13
12	$ssDS=[DS]-[D]-[S]+[Y]$	$(p-1)(t-1)$	13
13	$ssSxbl(D)=[DSR]-[DS]-[DR]+[D]$	$p(n-1)(t-1)$	N.A.
14	$ssVB=[VB]-[V]-[B]+[Y]$	$(q-1)(r-1)$	16
15	$ssDVB=[DVB]-[DV]-[DB]-[VB]+[D]+[V]+[B]-[Y]$	$(p-1)(q-1)(r-1)$	16
16	$ssVxBxbl(D)=[DVBR]-[DVB]-[DVR]-[DBR]+[DV]+[DB]+[DR]-[D]$	$p(n-1)(q-1)(r-1)$	N.A.
17	$ssVS=[VS]-[V]-[S]+[Y]$	$(q-1)(t-1)$	19
18	$ssDVS=[DVS]-[DV]-[DS]-[VS]+[D]+[V]-[Y]$	$(p-1)(q-1)(t-1)$	19
19	$ssVxSxbl(D)=[DVSR]-[DVS]-[DVR]-[DSR]+[DV]+[DS]+[DR]-[D]$	$p(n-1)(q-1)(t-1)$	N.A.
20	$ssBS=[BS]-[B]-[S]+[Y]$	$(r-1)(t-1)$	22
21	$ssDBS=[DBS]-[DB]-[DS]-[BS]+[D]+[B]+[S]-[Y]$	$(p-1)(r-1)(t-1)$	22
22	$ssBxSxbl(D)=[DBSR]-[DBS]-[DBR]-[DSR]+[DB]+[DS]+[DR]-[D]$	$p(n-1)(r-1)(t-1)$	N.A.
23	$ssVBS=[VBS]-[VB]-[VS]-[BS]+[V]+[B]+[S]-[Y]$	$(q-1)(r-1)(t-1)$	25
24	$ssDVBS=[DVBS]-[DVB]-[DVS]-[DBS]-[VBS]+[DV]+[DB]+[DS]+[VB]+[VS]+[BS]-[D]-[V]-[B]-[S]+[Y]$	$(p-1)(q-1)(r-1)(t-1)$	25
25	$ssVxBxSxbl(D)=[DVBSR]-[DVBS]-[DVBR]-[DVSR]-[DBSR]+[DVB]+[DVS]+[DBS]+[DVR]+[DBR]+[DSR]-[DV]-[DB]-[DS]-[DR]+[D]$	$p(n-1)(q-1)(r-1)(t-1)$	N.A.
26	$ssTO=[DVBSR]-[Y]$	$npqrt-1$	N.A.

Note: 1)D, V, B, S are fixed effects; and blocks are random  
2)"ss" refers to sums of squares and "bl" refers to blocks



Table 3.4 -- Sums of Squares for Split-Plot Factorial Design<sup>3</sup>

$[D] = \sum_{j=1}^p \frac{\left( \sum_{i=1}^n \sum_{k=1}^q \sum_{l=1}^r \sum_{m=1}^t Y_{ijklm} \right)^2}{nqrt}$	$[DVS] = \sum_{j=1}^p \sum_{k=1}^q \sum_{m=1}^t \frac{\left( \sum_{i=1}^n \sum_{l=1}^r Y_{ijklm} \right)^2}{nr}$
$[V] = \sum_{k=1}^q \frac{\left( \sum_{i=1}^n \sum_{j=1}^p \sum_{l=1}^r \sum_{m=1}^t Y_{ijklm} \right)^2}{nprt}$	$[DBS] = \sum_{j=1}^p \sum_{l=1}^r \sum_{m=1}^t \frac{\left( \sum_{i=1}^n \sum_{k=1}^q Y_{ijklm} \right)^2}{nq}$
$[B] = \sum_{l=1}^r \frac{\left( \sum_{i=1}^n \sum_{j=1}^p \sum_{k=1}^q \sum_{m=1}^t Y_{ijklm} \right)^2}{npqt}$	$[VBS] = \sum_{k=1}^q \sum_{l=1}^r \sum_{m=1}^t \frac{\left( \sum_{i=1}^n \sum_{j=1}^p Y_{ijklm} \right)^2}{np}$
$[S] = \sum_{m=1}^t \frac{\left( \sum_{i=1}^n \sum_{j=1}^p \sum_{k=1}^q \sum_{l=1}^r Y_{ijklm} \right)^2}{npqr}$	$[DVR] = \sum_{i=1}^n \sum_{j=1}^p \sum_{k=1}^q \frac{\left( \sum_{l=1}^r \sum_{m=1}^t Y_{ijklm} \right)^2}{rt}$
$[DV] = \sum_{j=1}^p \sum_{k=1}^q \frac{\left( \sum_{i=1}^n \sum_{l=1}^r \sum_{m=1}^t Y_{ijklm} \right)^2}{nrt}$	$[DBR] = \sum_{i=1}^n \sum_{j=1}^p \sum_{l=1}^r \frac{\left( \sum_{k=1}^q \sum_{m=1}^t Y_{ijklm} \right)^2}{qt}$
$[DB] = \sum_{j=1}^p \sum_{l=1}^r \frac{\left( \sum_{i=1}^n \sum_{k=1}^q \sum_{m=1}^t Y_{ijklm} \right)^2}{nqt}$	$[DSR] = \sum_{i=1}^n \sum_{j=1}^p \sum_{m=1}^t \frac{\left( \sum_{k=1}^q \sum_{l=1}^r Y_{ijklm} \right)^2}{qr}$
$[DS] = \sum_{j=1}^p \sum_{m=1}^t \frac{\left( \sum_{i=1}^n \sum_{k=1}^q \sum_{l=1}^r Y_{ijklm} \right)^2}{nqr}$	$[DVBS] = \sum_{j=1}^p \sum_{k=1}^q \sum_{l=1}^r \sum_{m=1}^t \frac{\left( \sum_{i=1}^n Y_{ijklm} \right)^2}{n}$
$[VB] = \sum_{k=1}^q \sum_{l=1}^r \frac{\left( \sum_{i=1}^n \sum_{j=1}^p \sum_{m=1}^t Y_{ijklm} \right)^2}{npt}$	$[DVBR] = \sum_{i=1}^n \sum_{j=1}^p \sum_{k=1}^q \sum_{l=1}^r \frac{\left( \sum_{m=1}^t Y_{ijklm} \right)^2}{t}$
$[VS] = \sum_{k=1}^q \sum_{m=1}^t \frac{\left( \sum_{i=1}^n \sum_{j=1}^p \sum_{l=1}^r Y_{ijklm} \right)^2}{npr}$	$[DVSR] = \sum_{i=1}^n \sum_{j=1}^p \sum_{k=1}^q \sum_{m=1}^t \frac{\left( \sum_{l=1}^r Y_{ijklm} \right)^2}{r}$
$[BS] = \sum_{m=1}^t \sum_{l=1}^r \frac{\left( \sum_{i=1}^n \sum_{j=1}^p \sum_{k=1}^q Y_{ijklm} \right)^2}{npq}$	$[DBSR] = \sum_{i=1}^n \sum_{j=1}^p \sum_{l=1}^r \sum_{m=1}^t \frac{\left( \sum_{k=1}^q Y_{ijklm} \right)^2}{q}$
$[DR] = \sum_{i=1}^n \sum_{j=1}^p \frac{\left( \sum_{k=1}^q \sum_{l=1}^r \sum_{m=1}^t Y_{ijklm} \right)^2}{qrt}$	$[DVSR] = \sum_{i=1}^n \sum_{j=1}^p \sum_{k=1}^q \sum_{l=1}^r \sum_{m=1}^t Y_{ijklm}$
$[DVB] = \sum_{j=1}^p \sum_{k=1}^q \sum_{l=1}^r \frac{\left( \sum_{i=1}^n \sum_{m=1}^t Y_{ijklm} \right)^2}{nt}$	$[Y] = \frac{\left( \sum_{i=1}^n \sum_{j=1}^p \sum_{k=1}^q \sum_{l=1}^r \sum_{m=1}^t Y_{ijklm} \right)^2}{npqrt}$

The equations in Table 3.3 are used to calculate the sums of squares for each main effect, and two, three, and four-way interaction and their respective error terms. The terms in each of the equations in Table 3.3 are defined in Table 3.4. Equations 3, 7, 10, 13, 16, 19, 22, and 25 in Table 3.3 are error mean squares for their respective main effects and interaction mean squares. The F-statistic, for each effect or interaction is then the ratio of its mean square term and the specified error mean square.

The F-statistic, effect degrees of freedom, and error degrees of freedom are used to calculate the P-value using the F distribution. The P-value is the probability that the effects measured in this program are only due to chance and not because the independent variable affects the dependent variable. Since the P-values in this study come from the F-distribution, the test of significance is also called an F-test. Equations 1 and 4 in Table 3.3 would be used to test the significance of a replications effect but such a test was not performed.

The above calculations were performed using SAS and the log transformed data. Type III sums of squares were used for the statistical tests. The SAS code used and output generated is shown in Appendix B. A summary of the results is shown in Table 3.5.

As mentioned previously, effects with a P-value less than 0.05 are deemed statistically significant. As can be seen in Table 3.5, all of the main effects and three of the two-way interactions were statistically significant. In fact, the three-way interaction of driver, vehicle, and surface (not shown) was significant at this level ( $F_{4,12}=3.82$ ,  $P=0.02$ ,  $MSE=0.0094$ ). It is especially interesting that this three-way interaction was statistically significant since there was only interaction between vehicle and surface and no\* interaction between driver and vehicle and between driver and surface. The principle of effect heredity is partly contradicted here. The principle of effect heredity states that an interaction that is composed of a weak main effect and a strong main effect is mostly due to the strong main effect<sup>8</sup>.

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\*Strictly speaking, it cannot be said that there is no interaction between driver and vehicle, only that an interaction cannot be proven to exist with this data.

Table 3.5 – Significance Tests of Main Effects and Two-Way Interactions on Natural Log Transformed Stopping Distance Data

Source of Variation	Effect df	Error df	Mean Square Error	F-statistic	P-value
Drivers	2	9	0.00939	11.1	0.0037
Vehicles	2	18	0.00403	89.6	0.0001
ABS	1	9	0.00409	154.2	0.0001
Surface	1	9	0.0123	54.3	0.0001
Drivers and Vehicles	4	18	0.00403	2.32	0.096
Drivers and ABS	2	9	0.00409	5.92	0.023
Drivers and Surface	2	9	0.0123	1.64	0.25
Vehicles and ABS	2	18	0.00443	8.51	0.0025
Vehicles and Surface	2	18	0.00246	4.46	0.027
ABS and Surface	1	9	0.00437	28.3	0.0005

However, the practical meaning and practical significance of this three-way interaction is obscure. As discussed below, this effect may not be statistically significant.

There are several assumptions and conditions that apply to the split-plot factorial analysis of variance<sup>3</sup>. These include a normally distributed response variable and homogeneity of response variance across the conditions being assessed in the different tests. The benefits of the natural log transformation of corrected stopping distance include reducing heterogeneity of variance and normalizing the data. Since the data is still somewhat skewed and there is some heterogeneity of variance, it is fortunate that the ANOVA procedures used in this study have been found to be robust to moderate violations of these assumptions.

Other necessary conditions of the ANOVA are that the subjects experience treatment levels in random order, which was done. It is assumed that the responses to the factors controlled in this study are independent, another necessary condition.

Beyond these, there is an assumption known as sphericity which holds only for within variables and interactions of within variables and interactions between a within-factor and the between-factor. This assumption states that between the levels of the factor or interaction, subject to random sampling error, variances are constant and covariances, differences between pairs of levels of a within factor, are constant for the set of observations taken. If this assumption is violated, the F-test tends to be less conservative. For example, an F-test with a nominal 0.05 level of significance ( $P\text{-value}=0.05$ ) might have an actual level of significance of 0.07 or 0.08 if the data does not meet the sphericity assumption. Tests of sphericity are not well supported in the SAS software used to analyze this data and so were not calculated. Instead, a Greenhouse-Geisser conservative F-test procedure<sup>3</sup> was used to assess the impact of this assumption on the present results.

The Greenhouse-Geisser conservative F-test procedure involves calculating conservative critical F-statistics. These conservative critical F-statistics are calculated by dividing both the effect and error degrees-of-freedom by the within-factor degrees of freedom<sup>3</sup>. For example, the conservative critical F-statistic for the test of the vehicle effect was determined with numerator degrees of freedom equal to  $(3-1)/(3-1) = 1$  and denominator degrees of freedom equal to  $3(4-1)(3-1)/(3-1) = 9$  and a  $P\text{-value} = 0.05$  level of significance, yielding a conservative critical F-statistic of 5.12 ( $F_{0.05; 1, 9} = 5.12$ ). Note that a within-factor with only two levels (or an interaction of two within factors of only two levels each) need not be assessed for compliance with the sphericity assumption because there is only one covariance between two levels. To put it more simply, the effect degree of freedom is already 1, and so cannot be made any smaller. Also note that the between factor drivers is not assessed in this way because the sphericity assumption does not apply to it.

For an interaction of a between-factor with a within-factor, the new critical-F value for the test of the Driver x ABS interaction was determined with numerator degrees of freedom equal to  $(3-1)(2-1)/(2-1) = 2$  and denominator degrees of freedom equal to  $3(4-1)(2-1)/(2-1) = (3)(3) = 9$  and a P-value = 0.05 level of significance, for a conservative critical F-statistic of 4.26 ( $F_{0.05\ 2, 9} = 4.26$ ).

The Greenhouse-Geisser conservative F-test procedure need not be performed on effects that were not found to be significant using the ANOVA. The procedure will yield a higher critical F-statistic when that is possible, so performing the procedure on effects already deemed insignificant is pointless.

Once the conservative critical F-statistics have been determined, they are compared to the calculated F-statistics. If the calculated F-statistics are greater than the conservative critical F-statistics, the effect is significant whether or not the sphericity assumption is met. If the calculated F-statistic is less than the conservative F-statistic but greater than the normal critical F-statistic ambiguity results and formal tests are necessary. Sphericity tests are then necessary to make a conclusive judgement. If sphericity tests are not feasible, either defer judgement (i.e., make no determination) or assume that moderate differences in variances and covariances are likely the result of sampling error only and not a cause for concern<sup>9</sup>.

Using this procedure vehicle, ABS, and surface effects, as well as driver-ABS, vehicle-ABS, vehicle-surface, and ABS-surface interactions were found to still be statistically significant. The conservative F-test brings doubt on the significance of the three-way interaction of drivers, vehicles, and surface. Unfortunately with available computing resources, it is not possible to make definitive statements about the significance of this interaction. Keep in mind that this procedure does not apply to driver effects and it was not applied to the interaction between drivers and vehicles and the interaction between drivers and surface and so those results remain unchanged. Table 3.6 shows the results of the procedure.

Table 3.6 – Greenhouse-Geisser Conservative F-Test

Source of Variation	Conservative Effect df	Conservative Error df	Conservative Critical F-statistic	Calculated F-statistic
Vehicles	1	9	5.12	89.6
ABS	1	9	5.12	154.2
Surface	1	9	5.12	54.3
Drivers and ABS	2	9	4.26	5.92
Vehicles and ABS	2	9	4.26	8.51
Vehicles and Surface	2	9	4.26	4.46
ABS and Surface	1	9	5.12	28.3

To answer the question whether all three drivers were statistically significantly different from one another or whether only some of the drivers were different from one another, multiple comparisons were performed. 95% confidence intervals were calculated on the difference of mean stopping distances between Drivers 1, 2, and 3 using the following equation for Tukey-Kramer multiple comparisons<sup>10</sup>:

$$95\% \text{ C.I.} = (\bar{Y}_i - \bar{Y}_{i+1}) \pm \frac{q_{\alpha, I, edof}}{\sqrt{2}} (\sqrt{MSE}) \sqrt{\frac{1^2}{n_i} + \frac{1^2}{n_{i+1}}}$$

Where:

$\bar{Y}_i$  and  $\bar{Y}_{i+1}$  are the driver mean stopping distances

$q_{\alpha, I, edof}$  is  $q$  critical at the  $\alpha = .05$  level, for 3 groups, and the error dof = 9

$MSE$  is the mean square error

$n_i$  and  $n_{i+1}$  are the number of stops of each driver

The driver mean stopping distances and mean square errors were taken from the SAS output shown in Appendix B. The differences between vehicles were also calculated this way. The multiple comparisons are shown in Table 3.7. A statistically significant difference with a probability of 95%

is indicated when the confidence interval does not include zero. Remember that these confidence intervals were calculated using log transformed data, so one should not draw conclusions about the size of the differences from these intervals.

Table 3.7 -- Multiple Comparisons of Natural Log Transformed Stopping Distance Data

Comparison	95% Confidence Interval
Driver 1 and 2	-0.02635, 0.08596
Driver 1 and 3	0.01555, 0.1279
Driver 2 and 3	-0.01334, 0.09714
Taurus and Town Car	-0.1316, -0.06439
Taurus and Neon	0.03205, 0.09855
Neon and Town Car	0.1299, 0.1967

As Table 3.7 shows, there was only a statistically significant difference between Drivers 1 and 3. There was not a statistically significant difference between Drivers 1 and 2 or between Drivers 2 and 3. In other words, it is certain that Driver 1 is different from Driver 3, but the difference between Driver 2 and Driver 1 and between Driver 2 and Driver 3 may be due to experimental noise.

For all of the vehicles, the differences were statistically significant. In other words, the differences between the Neon, Taurus, and Town Car were all greater than the experimental noise.

A multiple linear regression was performed on the data using Minitab. The model for the linear regression included all the main effects and two-way interactions that were previously found to be statistically significant (except the interaction between vehicles and surface)\*. The adjusted squared multiple correlation coefficient was calculated to be 0.73. This indicates that 73% of the variation

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\*Minitab automatically removed this interaction from the model because it was too closely correlated with other elements in the model.

in the stopping distances is explained by the elements in the model<sup>9</sup>. That is, 27% of the variation in stopping distance is not explained by differences between drivers, vehicles, ABS condition, and surface condition and their interactions. Thus, a fairly large amount of variation is not being controlled by the experimental method. The likely sources of this variation include environmental condition changes, test surface friction changes, brake system changes, etc.

Removing drivers and the interaction between drivers and ABS from the regression model reduces the percentage of explained variation in stopping distance to 66%. The 7% reduction in explained variation due to removing driver effects from the model is a measure the driver effect. Compared to the other independent factors, the interactions of the other independent factors, and the uncontrolled variability, driver effect is quite small.

Since the multiple linear regression does not use the split-plot factorial design, it is a less powerful discriminator and was not used for inferential tests of the independent factors.

### **3.2 Taurus Problems**

Testing was stopped in the middle of the third replication because the left rear brake on the Taurus started dragging. The rear brakes were disassembled and inspected. Inspection revealed that the rear brakes had been reassembled incorrectly during the brake rebuild at the beginning of the program.

Because the Taurus has four wheel disk brakes, the rear brakes are complicated by the parking brake. The parking brake is actuated by a rod threaded into the rear of the caliper piston. To prevent the piston from rotating with the rod, a slot in the piston face is supposed to engage with a pin in the brake pad backing plate. During a brake rebuild the installer must rotate the piston so that the slot in the piston aligns with the pin on the backing plate and the whole backing plate rests on the face of the piston.



This step was not performed during the initial brake rebuild. Since the pin was not aligned with the slot, the pad did not lie flat on the piston face and parallel to the brake disk surface. Instead, the pad contacted the piston face on the pin and part of the backing plate. This caused only part of the pad to contact the disk surface. During the brake burnish, the pad friction material wore away in a manner that brought most of the friction material into contact with the disk surface. See Figure 3.6. The dark area is the wear surface. The light colored area above the dark area is the portion of the friction material that was not contacting the rotor. The light colored area at each end is where the friction material was beveled at both ends at the factory.



Figure 3.6 – Taurus Brake Pad

To complete testing, the caliper pistons were retracted until the brake no longer dragged. Care was taken to reassemble the brakes with the pin and slot still misaligned, so that only the dragging condition was eliminated. Fixing the pin alignment problem at this point would have caused new

wear patterns of the pad and changed the brake operation. This could possibly make the data collected in replications 1 and 2 incomparable to data collected in replications 3 and 4. The vehicle was then run on the roller dynamometer. The roller dynamometer data indicated that the brakes were apparently operating normally and testing was resumed. Replications 3 and 4 were then completed.

Several steps were taken to verify that the Taurus data collected in the test program was representative of a Taurus with correctly functioning brakes. After replications 3 and 4 were completed, the rear brakes were rebuilt with new pads and with the pins and slots correctly aligned. The brake system was then burnished again according to FMVSS 135.

The vehicle was again run on the roller dynamometer. The roller dynamometer showed that there was not an appreciable difference in the operation of the brake system of the Taurus before and after correcting the problem with the brakes.

To further test the comparability of the data collected in the program with that of a Taurus with correctly rebuilt brakes, a fifth replication with all three drivers was run. The shortest stops from the first four replications was used to calculate a 95% confidence interval for the mean shortest stop for each driver and test condition. See Table 3.8. Confidence intervals calculated on the natural log transformed data and the natural log of the shortest stop from the fifth replication are presented. As can be seen, the shortest stop for each driver and test condition from the fifth replication was within 10 of the 12 confidence intervals. For driver 1, on dry asphalt, ABS-off, the shortest stop from the fifth replication was approximately one meter outside the confidence interval. However, this confidence interval was only calculated on 3 replications because in replication 4, the driver failed to achieve a stop without wheel lock-up. In replication 5 on wet asphalt, and ABS-off, driver 1 again did not get a stop without lock up.

As with most vehicles, the Taurus brakes are biased toward the front axle. Also, the rear brake pads had worn sufficiently during the first burnish, that most of the friction material was in contact with the rotor. Also, the least experienced driver had problems achieving a stop without wheel lock-up

Table 3.8 – 95% Confidence Interval From Replications 1-4 and Replication 5

Driver	Surface	ABS	Confidence Interval	Replication 5
1	dry	on	3.884 , 4.010	3.885
1	dry	off	4.013 , 4.090	4.126
1	wet	on	3.831 , 4.168	3.927
1	wet	off	3.984 , 4.268	NA
2	dry	on	3.854 , 3.932	3.868
2	dry	off	3.960 , 4.092	4.021
2	wet	on	3.897 , 4.023	3.913
2	wet	off	4.076 , 4.322	4.149
3	dry	on	3.809 , 3.941	3.855
3	dry	off	3.907 , 4.051	4.009
3	wet	on	3.802 , 4.005	3.900
3	wet	off	3.952 , 4.245	4.132

both before and after the brakes were repaired. Due to schedule and cost constraints, re-testing was not convenient or apparently necessary. Based on the evidence given above, the results for the test program for the Taurus were concluded to be valid .

#### **4.0 CONCLUSIONS**

All the main effects and most two-way interactions were statistically significant. This is somewhat remarkable given the relatively small effect sizes measured. The high level of statistical significance is due to the power of the experimental design used and the amount of data recorded rather than the size of the effects.

In fact, the size of the effect for drivers over all the vehicles and test conditions, was about five meters, only about a car length. However, the range of driver effect for a vehicle in a given test condition after four replications was from about 1 meter up to 36 meters over all the vehicles and test conditions. The worst case of driver difference (36 meters) was between Driver 1 and Driver 3 in the Town Car with the ABS-off on wet asphalt. This huge difference is atypical though, as is indicated by the only five meter difference between drivers over all the vehicles, test conditions, and replications.

The difference between the Taurus and the Neon and the difference between the Taurus and Town Car were of the same order of magnitude as the difference between Drivers 1 and 3. This indicates that there is a floor to the resolution of differences between vehicles that can meaningfully be measured using only one driver and one replication. Because the difference between ABS conditions is greater than the difference between drivers, a test method could be selected with high enough resolution to distinguish the benefits of ABS.

Requiring several replications in the test procedure could significantly limit driver effect. Looking at the data from this experiment, one can see that in most cases, after four replications, the range between each driver's shortest stopping distance is pretty small. Taking each driver's best stop from four replications in a particular vehicle and test condition, one can create what could be called a "best stop range". The average of the best stop ranges over all the vehicles and test conditions (excluding the Town Car with the ABS-off on wet asphalt) is 3.5 meters. This shows that after four replications, experienced drivers might be expected to get within 3.5 meters of each other. The implication of this is that if a test procedure used only one driver for each vehicle, with enough replications, the effect of that driver could be reduced to only 3.5 meters with few exceptions.

Given the amount of noise in current brake test methods, if a test procedure was used that only required one driver and one replication, the benefit of ABS might not be seen unless stops on a wet surface are also performed. The largest two-way interaction was between ABS condition and surface condition. The effect of surface condition went from 5 meters with ABS-on to 10 meters with ABS-off. This indicates that ABS is more important on wet asphalt than dry asphalt. If stops on wet asphalt were included in a test procedure for light vehicles, those vehicles with ABS would likely score better than those without.

The practical significance of the remaining two-way interactions is probably small. The largest of these effects is about 4 meters and is from the interaction of ABS condition and driver. Surprisingly, the most experienced driver was most affected by turning off the ABS.

Driver 3 was least affected by the presence of ABS. The next largest interactions were only 3 meters and were the interactions between vehicle and ABS condition and the interaction between vehicle and surface.

The fact that there is no strong interaction between driver and vehicle or surface is promising for the development of a test procedure for a light vehicle braking program. This shows that experienced drivers were not greatly affected by different size vehicles. It also shows that experienced drivers were not greatly affected by large changes in surface friction. This would mean that the smaller differences in test surfaces between test facilities would not amplify driver differences.

The multiple linear regression model discussed earlier explained 73% of the variation in stopping distances. This shows that the current brake testing methods have not completely controlled all the factors that affect brake tests. The uncontrolled variation is 27%. The likely sources of this variation include environmental condition changes, test surface friction changes, brake system changes, etc. Another source of this variation comes from random variability within drivers. Drivers in this study were modeled as fixed effects, but this is obviously a large simplification. Any test procedure adopted for an light vehicle braking program will have to take into account, that on top of the small but unwanted differences between drivers, there is quite a lot of variability from other sources.

In conclusion, a test procedure can be developed that accurately measures differences between vehicles. The test procedure used in this test program measured differences between all three of the vehicles tested with very high statistical certainty. Unfortunately, this test procedure also used a large number of replications and drivers.

However, despite the wide differences in mass of the vehicles used in this program, they may have fairly uniform braking performance compared to all the light vehicles on the road. Each of the test vehicles is probably equipped with the best brake package offered for that model. The Taurus used was a higher trim line, the Neon used was a “Sport” model, the Town Car is an expensive luxury car. They all had 4 wheel disk brakes with ABS.

A wider variety of vehicles were tested using similar test methods by VRTC in 1997. The range of stopping distances for the 1997 test vehicles with the ABS-on and on dry asphalt was 25%<sup>1</sup>. The range of stopping distances for the 1998 vehicles in this test condition was only 9%. The range of stopping distances for the 1997 test vehicles with the ABS-on and on wet asphalt was 30%. The range of stopping distances for the 1998 vehicles in this test condition was only 11%. Range is defined as the difference between the shortest stop over all the drivers and replications of the best (shortest) vehicle and the shortest stop of the worst (longest) vehicle divided by the shortest stop of the worst vehicle.

If the light vehicle brake test procedure need not be able to differentiate between every vehicle, a much smaller number of replications and drivers may be needed. If the goal of the light vehicle braking performance test program is only to distinguish large differences in braking performance, fewer replications and drivers would be needed for each vehicle tested.

## **APPENDIX A**

### **TEST VEHICLE INFORMATION**

## VEHICLE INFORMATION

Manufacturer: Ford Motor Co.	Model: Lincoln Town Car
Body style: 4 door sedan	VIN: 1LNLM82W6VY755238
Date of mfg: 8/97	Odometer: 1,810 (km)
Wheelbase: 2984 (mm)	Track front: 1613 (mm) rear: 1638 (mm)
GVWR: 2425 (kg)	GAWR front: 1156 (kg) rear: 1289 (kg)
Dates tested: 4/5/98-5/14/98	

### DRIVE TRAIN

Fuel type: gasoline Displacement: 4.6 (l) Number cylinders: 8  
Transmission: automatic Forward speeds: 3

### TIRES

Manufacturer: Michelin	Model: XW4
Size: P225/60R16	Pressure front: 220 (kpa) rear: 241 (kpa)

### BRAKES

ABS manufacturer: ITT	ABS type: 4 sensor, 4 circuit
Variable proportioning valve: none	
Master cylinder circuit split type: diagonal	
Booster type: vacuum	Parking brake control: foot
Front brake type: disk	Lining code: 6051 EE
Rear brake type: disk	Lining code: PFC 9FF 7050

### VEHICLE WEIGHT - All weights are with driver and full fuel tanks.

LLVW front: 1075 (kg) rear: 866 (kg)  
GVWR front: 1148 (kg) rear: 1279 (kg)



## VEHICLE INFORMATION

Manufacturer: Ford Motor Co.	Model: Taurus
Body style: 4 door sedan	VIN: 1FALP52UOTG132036
Date of mfg: 9/95	Odometer: 47,300 (km)
Wheelbase: 2750 (mm)	Track front: 1565 (mm) rear: 1560 (mm)
GVWR: 2135 (kg)	GAWR front: 1201 (kg) rear: 982 (kg)
Dates tested: 4/5/98-5/14/98	

### DRIVE TRAIN

Fuel type: gasoline Displacement: 3.0 (l) Number cylinders: 6  
Transmission: automatic Forward speeds: 3 speed

### TIRES

Manufacturer: American General	Model: G45
Size: P205/65R15	Pressure front: 227 (kpa) rear: 227 (kpa)

### BRAKES

ABS manufacturer: Bosch	ABS type: 4 sensor, 4 circuit
Variable proportioning valve: none	
Master cylinder circuit split type: diagonal	
Booster type: vacuum	Parking brake control: foot
Front brake type: disk	Lining code: AKNS171H-FF
Rear brake type: disk	Lining code: 292304 NT8-FF and 294671 NT8-FF

### VEHICLE WEIGHT - All weights are with driver and full fuel tanks.

LLVW front: 1057 (kg) rear: 653 (kg)  
GVWR front: 1170 (kg) rear: 966 (kg)

## VEHICLE INFORMATION

Manufacturer: Chrysler Corporation Model: Neon Sport Coupe  
Body style: 2 door coupe VIN: 1P3ES62C2SD167756  
Date of mfg: 10/94 Odometer: 12,461 (km)  
Wheelbase: 2635 (mm) Track front: 1455 (mm) rear: 1460 (mm)  
GVWR: 1595 (kg) GAWR front: 889 (kg) rear: 741 (kg)  
Dates tested: 4/5/98-5/14/98

### DRIVE TRAIN

Fuel type: gasoline Displacement: 2.0 (l) Number cylinders: 4  
Transmission: manual Forward speeds: 5

### TIRES

Manufacturer: Goodyear Model: Eagle RS-A  
Size: P185/65R14 Pressure front: 220 (kpa) rear: 220 (kpa)

### BRAKES

ABS manufacturer: Allied-Signal ABS type: 4 sensor, 4 circuit  
Variable proportioning valve: none  
Master cylinder circuit split type: diagonal  
Booster type: vacuum Parking brake control: hand  
Front brake type: disk Lining code: 6050 EE 56777.8093  
Rear brake type: disk Lining code: 7050 BX HJ FF 322

### VEHICLE WEIGHT - All weights are with driver and full fuel tanks.

LLVW front: 771 (kg) rear: 463 (kg)  
GVWR front: 866 (kg) rear: 726 (kg)

## **APPENDIX B**

### **SAS CODE AND OUTPUT**

Below is the SAS code used to analyze the data for significance of the main effects and interactions.  
See Table 3.1 for the symbols used for the independent factors.

```
libname sas 'd:\thesis\data';
filename txt 'd:\thesis\data';
data sas.ssd;
    infile txt('ssd.txt');
    input r 7 d $ 1-5 v $ 9-15 b $ 17-19 s $ 21-23 SD 27-31 logSD 33-37;
proc sort data=sas.ssd;
    by r d v b s;
proc glm;
    class r d v b s;
    model logSD=
        d r(d)
        v d*v v*r(d)
        b d*b b*r(d)
        s d*s s*r(d)
        v*b d*v*b v*b*r(d)
        v*s d*v*s v*s*r(d)
        b*s d*b*s b*s*r(d)
        v*b*s d*v*b*s v*b*s*r(d);
    test h=d          e=r(d);
    test h=v          e=v*r(d);
    test h=d*v        e=v*r(d);
    test h=b          e=b*r(d);
    test h=d*b        e=b*r(d);
    test h=s          e=s*r(d);
    test h=d*s        e=s*r(d);
    test h=v*b        e=v*b*r(d);
    test h=d*v*b      e=v*b*r(d);
    test h=v*s        e=v*s*r(d);
    test h=d*v*s      e=v*s*r(d);
    test h=b*s        e=b*s*r(d);
    test h=d*b*s      e=b*s*r(d);
    test h=v*b*s      e=v*b*s*r(d);
    test h=d*v*b*s    e=v*b*s*r(d);
    means d v b s d*v d*b d*s/T lines;
run;
```

Below is the output generated by SAS:

The SAS System

General Linear Models Procedure  
Class Level Information

Class	Levels	Values
R	4	1 2 3 4
D	3	Bob Larry Lyle
V	3	Taurus TownCar neon
B	2	off on
S	2	Dry Wet

Number of observations in data set = 141

General Linear Models Procedure

Dependent Variable: LOGSD

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	140	2.8011608	0.0200083	.	.
Error	0	.	.		
Corrected Total	140	2.8011608			

R-Square	C.V.	Root MSE	LOGSD Mean
1.000000	0	0	5.2016

Source	DF	Type I SS	Mean Square	F Value	Pr > F
D	2	0.1210488	0.0605244	.	.
R(D)	9	0.0564542	0.0062727	.	.
V	2	0.6415163	0.3207582	.	.
D*V	4	0.0225851	0.0056463	.	.
R*V(D)	18	0.0786070	0.0043671	.	.
B	1	0.5383667	0.5383667	.	.
D*B	2	0.0287053	0.0143526	.	.
R*B(D)	9	0.0352110	0.0039123	.	.
S	1	0.6328661	0.6328661	.	.
D*S	2	0.0258663	0.0129331	.	.
R*S(D)	9	0.1182707	0.0131412	.	.
V*B	2	0.0613032	0.0306516	.	.
D*V*B	4	0.0152283	0.0038071	.	.
R*V*B(D)	18	0.0922010	0.0051223	.	.
V*S	2	0.0160690	0.0080345	.	.
D*V*S	4	0.0416963	0.0104241	.	.
R*V*S(D)	18	0.0328395	0.0018244	.	.
B*S	1	0.1075022	0.1075022	.	.
D*B*S	2	0.0165945	0.0082973	.	.
R*B*S(D)	9	0.0553849	0.0061539	.	.
V*B*S	2	0.0043688	0.0021844	.	.
D*V*B*S	4	0.0164866	0.0041217	.	.
R*V*B*S(D)	15	0.0419891	0.0027993	.	.

Source	DF	Type III SS	Mean Square	F Value	Pr > F
D	2	0.2084020	0.1042010	.	.
R(D)	9	0.0844703	0.0093856	.	.
V	2	0.7223002	0.3611501	.	.
D*V	4	0.0374045	0.0093511	.	.
R*V(D)	18	0.0725216	0.0040290	.	.
B	1	0.6303076	0.6303076	.	.
D*B	2	0.0483777	0.0241888	.	.
R*B(D)	9	0.0367835	0.0040871	.	.
S	1	0.6706536	0.6706536	.	.
D*S	2	0.0404555	0.0202278	.	.
R*S(D)	9	0.1110756	0.0123417	.	.
V*B	2	0.0754176	0.0377088	.	.
D*V*B	4	0.0178627	0.0044657	.	.
R*V*B(D)	18	0.0798021	0.0044334	.	.
V*S	2	0.0219119	0.0109559	.	.

Source	DF	Type III SS	Mean Square	F Value	Pr > F
D*V*S	4	0.0375019	0.0093755	.	.
R*V*S(D)	18	0.0442015	0.0024556	.	.
B*S	1	0.1236549	0.1236549	.	.
D*B*S	2	0.0215856	0.0107928	.	.
R*B*S(D)	9	0.0393398	0.0043711	.	.
V*B*S	2	0.0080930	0.0040465	.	.
D*V*B*S	4	0.0164866	0.0041217	.	.
R*V*B*S(D)	15	0.0419891	0.0027993	.	.

# General Linear Models Procedure

Dependent Variable: LOGSD

Tests of Hypotheses using the Type III MS for R(D) as an error term

Source	DF	Type III SS	Mean Square	F Value	Pr > F
D	2	0.2084020	0.1042010	11.10	0.0037

Tests of Hypotheses using the Type III MS for R\*V(D) as an error term

Source	DF	Type III SS	Mean Square	F Value	Pr > F
V	2	0.7223002	0.3611501	89.64	0.0001

Tests of Hypotheses using the Type III MS for R\*V(D) as an error term

Source	DF	Type III SS	Mean Square	F Value	Pr > F
D*V	4	0.0374045	0.0093511	2.32	0.0962

Tests of Hypotheses using the Type III MS for R\*B(D) as an error term

Source	DF	Type III SS	Mean Square	F Value	Pr > F
B	1	0.6303076	0.6303076	154.22	0.0001

Tests of Hypotheses using the Type III MS for R\*B(D) as an error term

Source	DF	Type III SS	Mean Square	F Value	Pr > F
D*B	2	0.0483777	0.0241888	5.92	0.0229

Tests of Hypotheses using the Type III MS for R\*S(D) as an error term

Source	DF	Type III SS	Mean Square	F Value	Pr > F
S	1	0.6706536	0.6706536	54.34	0.0001

Tests of Hypotheses using the Type III MS for R\*S(D) as an error term

Source	DF	Type III SS	Mean Square	F Value	Pr > F
D*S	2	0.0404555	0.0202278	1.64	0.2472

Tests of Hypotheses using the Type III MS for R\*V\*B(D) as an error term

Source	DF	Type III SS	Mean Square	F Value	Pr > F
V*B	2	0.0754176	0.0377088	8.51	0.0025

Tests of Hypotheses using the Type III MS for R\*V\*B(D) as an error term

Source	DF	Type III SS	Mean Square	F Value	Pr > F
D*V*B	4	0.0178627	0.0044657	1.01	0.4296

Tests of Hypotheses using the Type III MS for R\*V\*S(D) as an error term

Source	DF	Type III SS	Mean Square	F Value	Pr > F
V*S	2	0.0219119	0.0109559	4.46	0.0267

Tests of Hypotheses using the Type III MS for R\*V\*S(D) as an error term

Source	DF	Type III SS	Mean Square	F Value	Pr > F
D*V*S	4	0.0375019	0.0093755	3.82	0.0204

Tests of Hypotheses using the Type III MS for R\*B\*S(D) as an error term

Source	DF	Type III SS	Mean Square	F Value	Pr > F
B*S	1	0.1236549	0.1236549	28.29	0.0005

Tests of Hypotheses using the Type III MS for R\*B\*S(D) as an error term

Source	DF	Type III SS	Mean Square	F Value	Pr > F
D*B*S	2	0.0215856	0.0107928	2.47	0.1397

Tests of Hypotheses using the Type III MS for R\*V\*B\*S(D) as an error term

Source	DF	Type III SS	Mean Square	F Value	Pr > F
V*B*S	2	0.0080930	0.0040465	1.45	0.2666

Tests of Hypotheses using the Type III MS for R\*V\*B\*S(D) as an error term

Source	DF	Type III SS	Mean Square	F Value	Pr > F
D*V*B*S	4	0.0164866	0.0041217	1.47	0.2598



The SAS System

General Linear Models Procedure

Level of		-----LOGSD-----	
D	N	Mean	SD
Bob	45	5.23617778	0.16808930
Larry	48	5.20639583	0.14369186
Lyle	48	5.16447917	0.09962803
Level of		-----LOGSD-----	
V	N	Mean	SD
Taurus	47	5.19191489	0.10433513
TownCar	46	5.28989130	0.14917733
neon	48	5.12656250	0.11901402
Level of		-----LOGSD-----	
B	N	Mean	SD
off	69	5.26320290	0.16165591
on	72	5.14262500	0.08490864
Level of		-----LOGSD-----	
S	N	Mean	SD
Dry	72	5.13945833	0.08935874
Wet	69	5.26650725	0.15650123
Level of	Level of	-----LOGSD-----	
D	V	N	SD
Bob	Taurus	15	0.08399700
Bob	TownCar	14	0.17768791
Bob	neon	16	0.18183691
Larry	Taurus	16	0.12137763
Larry	TownCar	16	0.15746852
Larry	neon	16	0.07435579
Lyle	Taurus	16	0.09717851
Lyle	TownCar	16	0.09476638
Lyle	neon	16	0.06004984

General Linear Models Procedure

Level of D	Level of B	N	Mean	SD
Bob	off	21	5.30404762	0.20549415
Bob	on	24	5.17679167	0.09718784
Larry	off	24	5.28316667	0.15120664
Larry	on	24	5.12962500	0.08383878
Lyle	off	24	5.20750000	0.11181040
Lyle	on	24	5.12145833	0.06260156

Level of D	Level of S	N	-----LOGSD-----	
			Mean	SD
Bob	Dry	24	5.16758333	0.09998561
Bob	Wet	21	5.31457143	0.19637479
Larry	Dry	24	5.13645833	0.08515101
Larry	Wet	24	5.27633333	0.15727010
Lyle	Dry	24	5.11433333	0.07686333
Lyle	Wet	24	5.21462500	0.09553639

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